# Unsteady waves generated by two ships with different speeds

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## HIGHLIGHTS

- Development of a methodology to predict the waves generated by two ships with different speeds.
- Development of an uncoupled method to predict the encountering, overtaking and crossing scenarios.

## 1 INTRODUCTION

In previous studies on ship-to-ship interaction problem (Yeung, 1978), within the framework of potential-flow theory, the so-called double-body flow was applied on the calm-water surface which can be expressed as

$$\frac{\partial \Phi}{\partial n} = 0 \ , \quad \text{on z=0} \tag{1}$$

where $\Phi$ is the unsteady perturbation potential. Since the forward speed $U$ is not included in the free surface boundary condition, the speed of the two vessels can be arbitrary. However, in many manoeuvring operations, the encountering or overtaking speed is actually very high (Froude number $F_n>0.2$), especially when the lateral separation between ships is large. Thus, far-field effects arising from ship waves can be very important. The hydrodynamic interaction model must take into account of the surface-wave effects. In the present study, the well-known linear but unsteady free-surface condition in the moving frame of the ship will be used:

$$\frac{\partial^2 \Phi}{\partial t^2} - 2U\frac{\partial^2 \Phi}{\partial x \partial t} + U^2\frac{\partial^2 \Phi}{\partial x^2} + g\frac{\partial \Phi}{\partial z} = 0 \ , \text{ on z=0} \tag{2}$$

where $g$ is the acceleration due to gravity with the influence of the forward speed $U$ included. During the overtaking or encountering process, the two ships have different speeds. Consequently, it is not straightforward to define the free-surface condition by a single equation like Eq.(2). One way of overcoming this is to have a globally fixed frame of reference but the body geometry will change in time. Alternatively, to account for the different speeds appearing on the free-surface boundary condition, we propose superposition formulation. This method will eventually be applied to a general multi-body hydrodynamic-interaction program "MHydro" (Yuan et al., 2015) to investigate the interactive forces and wave patterns between two ships with different speeds.

## 2 SUPERPOSITION PRINCIPLE OF POTENTIAL

Consider, in general, $N$ ships moving at speeds $U_j$ ($j$ = 1, 2, ..., $N$) with respect to a space-fixed reference frame $\mathbf{x} = (x, y, z)$ in an inviscid fluid of depth $h$. A right-handed local Cartesian coordinate system $\mathbf{x_j} = (x_j, y_j, z_j)$ ($j$ = 1, 2, ..., $N$) is fixed to each ship with its positive $x_j$-direction pointing towards the bow, positive $z$-direction pointing upwards and $z_j$ = 0 is the undisturbed free-surface. Let $\Phi$ ($\mathbf{x}$, $t$) be the velocity potential describing the disturbances due to the forward motion of the ships and $\zeta$ ($x$, $y$, $t$) be the free-surface elevation. Assuming the disturbance of the fluid is small, we represent the total velocity potential produced by the presence of all ships in the fluid domain in a space-fixed frame to satisfy the following superposition principle:

$$\Phi(\mathbf{x},t) = \sum_{j=1}^{N} \Phi_j(\mathbf{x},t) \ , \ j = 1, 2, ..., N \tag{3}$$

where $\Phi_j$ ($\mathbf{x}$, $t$) is the velocity potential produced by the movement of ship $j$ at speed $U_j$, while the remaining ships are momentarily stationary in this frame. For the linear problem, the body-fixed coordinate system $\mathbf{x_j} = (x_j, y_j, z_j)$ ($j$ = 1, 2, ..., $N$) is used to solve the BVP for each of the $N$ vessels in concurrent motion. The relation between the body- and space-fixed coordinate system is straightforward, viz.

$$x_j = x - U_j t \ , \ j = 1, 2, ..., N \tag{4}$$

Let $\phi_j$ ($\mathbf{x_j}$, $t$) represents $\Phi_j$ ($\mathbf{x}$, $t$) in the *body-fixed* coordinate system, the following time-derivative can be obtained

$$\frac{d\Phi_j}{dt} = \left( \frac{\partial}{\partial t} - U_j \frac{\partial}{\partial x_j} \right) \phi_j \tag{5}$$

The velocity potential $\phi_j$ satisfies the Laplace equation and the body 'exact' boundary condition:

$$\nabla^2 \phi_j(\mathbf{x_j}, t) = 0 \,,\ j = 1, 2, \ldots, N \tag{6}$$

$$\frac{\partial \phi_j}{\partial n} = \delta_{ij} U_j \left( n_x \right)_j \,, \text{ on } \mathscr{B}_i, \text{ where } i, j = 1, 2, \ldots, N \tag{7}$$

where $\delta_{ij}$ is the Kronecker delta. The linearized free-surface condition (LFC) in the *j-th body-fixed* coordinate system can be obtained

$$\frac{\partial^2 \phi_j}{\partial t^2} - 2U_j \frac{\partial^2 \phi_j}{\partial x_j \partial t} + U_j^2 \frac{\partial^2 \phi_j}{\partial x_j^2} + g \frac{\partial \phi_j}{\partial z_j} = 0 \,, \text{ on } z = 0 \tag{8}$$

The boundary condition on the sea bottom and any side walls, if present, can be expressed as

$$\frac{\partial \phi_j}{\partial n} = 0 \tag{9}$$

Besides, a radiation condition is imposed on the control surface to ensure that waves vanish at upstream infinity

$$\phi_j \to 0, \ \ \zeta_j \to 0 \ \ as \ \sqrt{x_j^2 + y_j^2} \to \infty \tag{10}$$

where $\zeta_j$ is the wave elevation as seen in the *j-th body-fixed* frame and is given by Eq. (16).
Eqs. (6)-(10) form a complete set of BVP. Each one of BVP is time-dependent but can be solved individually and independently; only a single speed of ship $j$ appears in the free-surface condition in Eq. (8). The coupled problem is decoupled into $N$ independent BVPs. At each time instant, a simple source (Rankine) source panel method (Bai and Yeung, 1974) is used to solve the BVP represented by Eqs. (6) to (10). The unsteady free surface boundary condition will be solved in time domain by an iteration scheme with respect to the time.
Once the unknown potential $\phi_j$ is solved on the plane z=0 and on the body $\mathscr{B}_j$, the unsteady pressure components under its individual coordinate system can be obtained from linearized Bernoulli's equation

$$p_j \Big|_{\mathbf{x_j}} = -\rho \left[ \frac{\partial \phi_j}{\partial t} \Big|_{\mathbf{x_j}} - U_j \frac{\partial \phi_j}{\partial x_j} \Big|_{\mathbf{x_j}} \right] \,,\ j = 1, 2, \ldots, N \tag{11}$$

We should point out that because of the first unsteady term in Eq. (11), the total pressure $p$ in coordinate system $\mathbf{x_j}$ cannot be expressed directly as the sum of all the pressure components in their local frames. To transfer the pressure from coordinate system $\mathbf{x_i}$ to $\mathbf{x_j}$, the following relation needs to be observed

$$\frac{d\phi_i}{dt} \Big|_{\mathbf{x_j}} = \left( \frac{\partial}{\partial t} - \left( U_j - U_i \right) \frac{\partial}{\partial x_i} \right) \phi_i \Big|_{\mathbf{x_j}} \,,\ i, j = 1, 2, \ldots, N \tag{12}$$

It should be noted that the partial derivative symbol of the first term in Eq. (11) is retained to make it consistent with Eq. (5) where the potential is expressed in the *body-fixed* coordinate system $\mathbf{x_j}$. But here the body-fixed coordinate system $\mathbf{x_j}$ turns out to be in the reference frame for the other body-fixed coordinate system $\mathbf{x_i}$. Therefore, $\frac{\partial \phi_j}{\partial t}$ is actually calculated as a total derivative by using Eq. (12). The unsteady pressure in coordinate system $\mathbf{x_i}$ ( $i = 1, 2, \ldots, N, i \neq j$ ) can then be *'transferred'* to $\mathbf{x_j}$ as

$$p_i \Big|_{\mathbf{x_j}} = -\rho \left[ \left( \frac{\partial}{\partial t} - \left( U_j - U_i \right) \frac{\partial}{\partial x_i} \right) \phi_i \Big|_{\mathbf{x_j}} - U_i \frac{\partial \phi_i}{\partial x_i} \Big|_{\mathbf{x_j}} \right] = -\rho \left( \frac{\partial}{\partial t} - U_j \frac{\partial}{\partial x_i} \right) \phi_i \Big|_{\mathbf{x_j}} \,,\ i, j = 1, 2, \ldots, N \tag{13}$$

Note the subtle differences between Eq. (11) and Eq. (13). The total pressure $p$ in coordinate system $\mathbf{x_j}$ can be written as

$$p \Big|_{\mathbf{x_j}} = \sum_{i=1}^{N} p_i \Big|_{\mathbf{x_j}} = -\rho \sum_{i=1}^{N} \left( \frac{\partial}{\partial t} - U_j \frac{\partial}{\partial x_i} \right) \phi_i \Big|_{\mathbf{x_j}} \,,\ i, j = 1, 2, \ldots, N \tag{14}$$

Integrating the pressure over the hull surface, we can express the forces (or moments) on the $i$-th hull induced by the $j$-th ship as:

$$F_k^j = \iint_{S_j} p n_k dS \,,\ j = 1, 2, \ldots, N \tag{15}$$

where $k = 1, 2, \ldots, 6$, representing the force in surge, sway, heave, roll, pitch and yaw directions. The free-surface elevation can be obtained from dynamic free-surface boundary condition. Similar to the pressure expression, the unsteady wave elevation in coordinate system $\mathbf{x_i}$ ( $i = 1, 2, \ldots, N, i \neq j$ ) can be transferred to $\mathbf{x_j}$ as

$$\zeta_i \big|_{\mathbf{x_j}} = -\frac{1}{g}\left(\frac{\partial}{\partial t} - U_j \frac{\partial}{\partial x_i}\right)\phi_i \big|_{\mathbf{x_j}}, \; i, j = 1, 2, \ldots, N \tag{16}$$

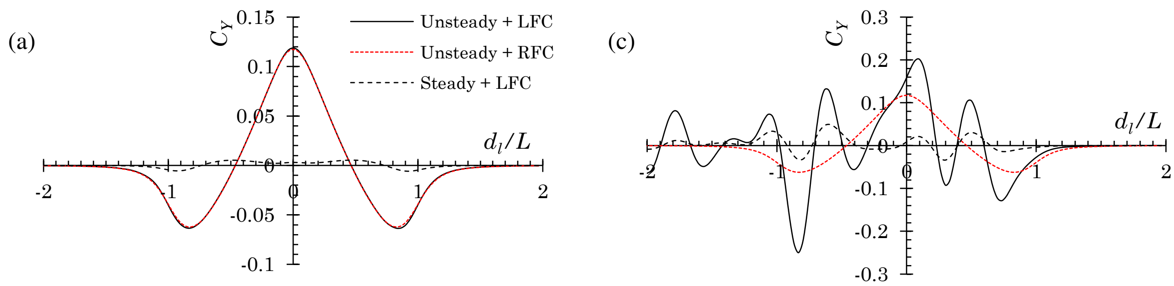
The total wave elevation in coordinate system $\mathbf{x_j}$ can be written as

$$\zeta \big|_{\mathbf{x_j}} = -\frac{1}{g}\sum_{i=1}^{N}\left(\frac{\partial}{\partial t} - U_j \frac{\partial}{\partial x_i}\right)\phi_i \big|_{\mathbf{x_j}}, \; i, j = 1, 2, \ldots, N \tag{17}$$

We note that we have not imposed a Kutta condition at the stern, as a first approximation, or equivalently, the stern is pointed.

## 3 RESULTS AND DISCUSSIONS

The results in Fig. 1 show the effects of unsteady pressure and unsteady free surface. Here, we note that the unsteady pressure term in Eq. (14) is very important at all the range of encountering speeds, while the free-surface effect is only important when the encounter speed is moderate or high. Ignoring the unsteady pressure term in Eq. (14) will lead to mis-estimation of the interaction force. At $F_n = 0.1$, the free-surface elevation and hydrodynamic interaction are mainly determined by the near-field (non-wave-like) disturbances. The rigid free-surface condition (RFC) is adequate to predict the interaction forces, as shown in Fig. 1. (a). As the Froude number increases to $F_n$ =0.2, the far-field waves become manifest, and the interaction force oscillates correspondingly, as shown in Fig. 1. (b). At $F_n$ =0.2, the interaction is still dominated by the near-field disturbance. The contribution of the force induced by far-field waves is smaller than that induced by the near-field disturbance. The free-surface effect becomes more significant at $F_n = 0.3$. The force amplitude induced by the far-field waves is larger than that induced by the near-field disturbance, as shown in Fig. 1(c). There are only three peaks induced by near-field disturbance. However, the peaks altered by the far-field waves are less obvious. It can be concluded that free-surface effects must be taken into account at $F_n > 0.2$.



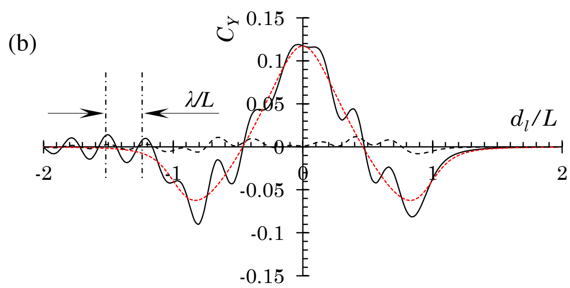




Fig. 1. Sway force acting on two identical Wigley III hulls in head-on encounter with $d_t/B$=2. (a) $F_n$=0.1; (b) $F_n$=0.2; (c) $F_n$=0.3. $d_l/L$=0 corresponds to the moment $t=t_s$, when the midships of the two ships are aligned. $d_l/L$>0 corresponds to $t<t_s$, $d_l/L$<0 corresponds to $t>t_s$. $C_Y$ is non-dimensionalized by $\frac{1}{2}\rho BT|U_1 U_2|$. LFC indicates that the linearized free-surface condition is used; RFC indicates that the rigid-wall free-surface condition is used.

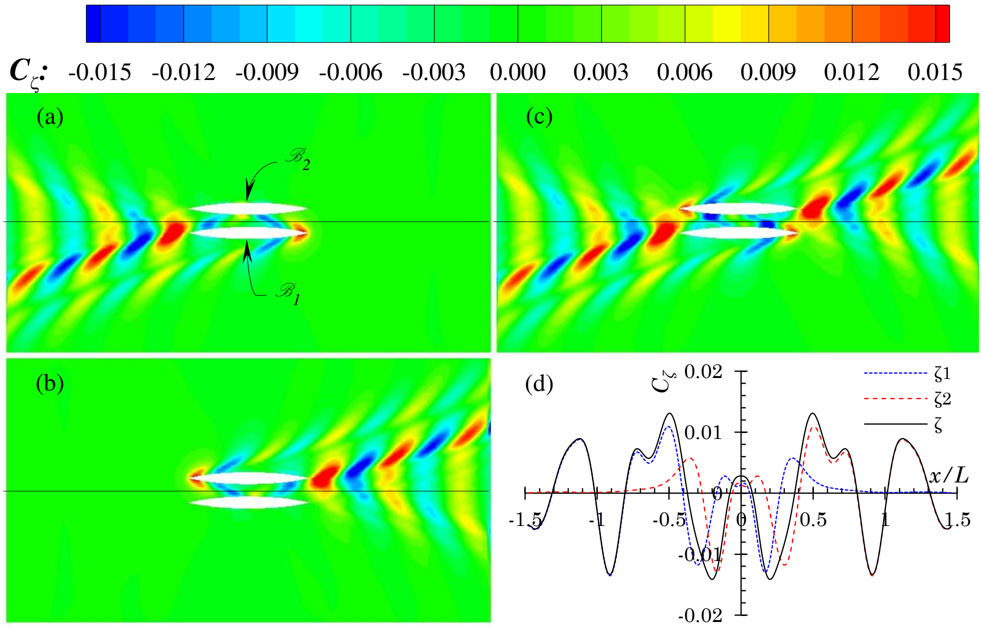



Fig. 2. Waves produced by two Wigley III (Journee, 1992) hulls of equal speed in head-on encounter with $d_t/B$=2, $d_l$=0 and $F_n$=0.3. (a) $C_{\zeta 1}$, the waves produced by $\mathscr{B}_1$ moving at $F_n$=0.3 while $\mathscr{B}_2$ is momentarily stationary in the body-fixed frame of $\mathscr{B}_1$; (b) $C_{\zeta 2}$, the waves produced by $\mathscr{B}_2$ moving at $U_2$ while $\mathscr{B}_1$ is momentarily stationary in the body-fixed frame of $\mathscr{B}_2$; (c) $C_\zeta$, the total waves superposing $C_{\zeta 1}$ and $C_{\zeta 2}$; (d) Wave profile at centre line between two hulls shown in (a), (b) and (c). $x$ in the abscissa of (d) refers to the midship-to-midship distance between left-moving ship and the encountered ship.

Fig. 2 shows the wave elevation components obtained by the present superposition principle. It should be noted that the total wave elevation presented in Fig. 2(c) is not the simple superposition of the waves produced by two individual hulls moving towards opposite direction. When we calculate the wave elevation produced by $\mathscr{B}_1$, the presence of $\mathscr{B}_2$ is also considered, treated as an obstacle, by being momentarily stationary in the body-fixed frame of $\mathscr{B}_1$. Therefore, the diffraction and reflection by $\mathscr{B}_2$ is considered in the present study. These reflected waves can be seen clearly from Fig. 2.(a) and (b).

## 4 CONCLUSIONS

The present study proposed a methodology to deal with the different-speed problem when the two ships are encountering or overtaking each other. The superposition principle was applied to decouple the boundary condition on the free surface. Numerical results indicate the near-field disturbance is the most importance component of the interaction force when the encountering speed is low. As the encountering speed increases, the interaction force induced by the far-field waves manifests gradually. It was found the free-surface effects must be considered at $F_n > 0.2$ for slender ships. For blunt ships, the lower limit of Froude number is smaller. When the encountering speed reaches $F_n$ = 0.3, the free-surface effect becomes the dominant component.

## REFERENCES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